

## A surface parameter-based method for accuracy and efficient tool path generation (Application of general curve on parametric space)

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### Abstract

This study deals with a new generation method of tool paths on the main processor in a CAM system. In general, when commercial CAMs generate a tool path, a contouring mode, a scanning mode, a constant scallop height mode, and other tool path generation methods can be selected. However, in the algorithms of path generation, the differential and convergence calculations, and the complex branch conditions according to the various surfaces become a bottleneck in calculation speed, and so the real-time property is problematical. Therefore, two parameters ( $u, v$ ) composing a surface are the focus of this study. The tool path is generated on the  $(u, v)$  plane, which maintains consistency with real space. This novel method of tool path generation does not experience the problems of singular points and complex branch conditions as compared with the general method. Moreover, we confirm that the calculation speed is dramatically improved for explicit functions on the  $(u, v)$  plane, as described in a previous report. In this paper, the function on the  $(u, v)$  plane is expanded to general curves expressed as parametric curves. The effectiveness of our method is improved by the use of parametric curves.

CAM, main processor, tool path generation

### 1. Introduction

Currently, the mainstream methods of tool path generation are a scanning path, a contouring path, or a composite of these paths. Various other methods, such as a spiral path and a constant scallop height path, are also available. Computer-aided manufacturing (CAM) operators can generate a tool path by selecting the optimum method with consideration of the features of the machining surface. However, all algorithms of tool path generation have the following problems.

- (1) Since a differential or a convergence calculation is required, the accuracy of the generated tool path is affected by the step size of the differential calculation or the convergence of the algorithm.
- (2) Since the features of several free surfaces and surface rank require complex branch conditions, the calculation time is long.

These problems also cause a lack of desired tool paths and failed computations.

This study proposes a novel method of tool path generation. As shown in Figure 1 (a), the conventional method calculates a path in real space, although this causes a surface/surface intersection problem [1]. Instead, our proposed method calculates a path in parametric space, which is composed of  $u$  and  $v$  parameters on a machining surface expressed as  $S(u, v)$ , as shown in Figure 1 (b). The proposed method can avoid the above problems of the features of the machining surface. Our previous report [2] referred to the generation method of a

homogeneous tool path that keeps the generation interval of machining points constant for a limited environment in which lines and circles are defined on the  $(u, v)$  plane. In this paper, our method is expanded to general curves expressed as parametric curves.

### 2. Tool path generation on $(u, v)$ plane

As shown in Figure 2, a free curve on the  $(u, v)$  plane is expressed as  $C(t)$  for a machining surface  $S(u, v)$ . The interval of machining points in real space is expressed as  $r$ . When  $r$  is minute, the following equation is obtained.

$$\|S_u du + S_v dv\| = dr \quad (1)$$

where  $S_u$  and  $S_v$  are first-order differential coefficients in the  $u$

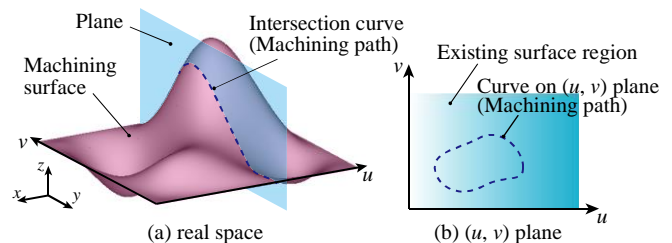


Figure 1. Comparison of tool path generation conventional (a) and proposed (b) method

and  $v$  directions on  $S$ , respectively. Also, when  $C_{t,u}$  and  $C_{t,v}$  are components of the  $u$  and  $v$  directions of the first-order differential coefficients at parameter  $t$  on  $C$ ,

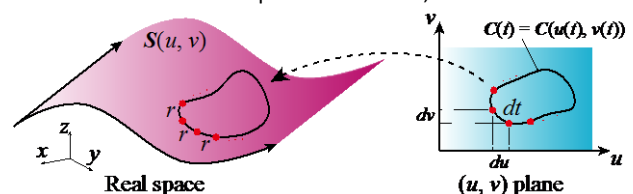


Figure 2. Relationship between a curve on  $(u, v)$  plane and real space

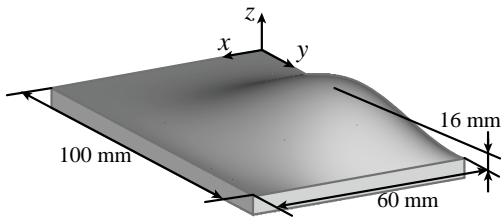


Figure 3. Defined workpiece for verification

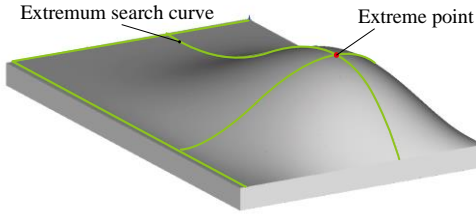


Figure 4. Generation of extremum search curve and detection of extreme point

Table 1. Specifications of tool path generation

Number of turns in a spiral	30
Interval of machining points [mm]	0.2
Number of control points of NURBS interpolation curve	121

$$\frac{dv}{du} = \frac{dv}{dt} / \frac{du}{dt} = \frac{C_{t,u}}{C_{t,v}} \quad (2)$$

is obtained. By substituting Eq. (3) into Eq. (2), the following equations are obtained.

$$\frac{du}{dr} = \frac{C_{t,u}}{\|C_{t,u}S_u + C_{t,v}S_v\|} \quad (3)$$

$$\frac{dv}{dr} = \frac{C_{t,v}}{\|C_{t,u}S_u + C_{t,v}S_v\|} \quad (4)$$

Since Eqs. (3) and (4) are ordinary differential equations, numerical calculation methods such as the Runge-Kutta method can be applied and machining points can be generated quickly at equal intervals in real space. However, since  $C_{t,u}$  and  $C_{t,v}$  depend on  $t$  in Eq. (4),  $t$  and  $(u, v)$  must be updated by the following equation.

$$dt = \frac{\sqrt{du^2 + dv^2}}{\|C_t\|} \quad (5)$$

### 3. Verification

We generated a spiral path with the proposed method. Figure 3 shows the defined free surface expressed as a non-uniform rational B-spline (NURBS) (rank = 3), and Table 1 shows the setting parameters for tool path generation. A workstation (OS: Windows 7 64 bit; CPU: Xeon 3.60 GHz, 8 cores; memory: 16 GB) was used for path generation; however, only a single CPU core was used for the calculations. The following is the generation process.

- i. A convergent point calculated by generating the extremum search curve [3] is obtained.
- ii. In the  $(u, v)$  plane, four corners in the defined region on the surface and the convergent point are connected by lines.
- iii. Each line is divided into a designated number of turns in a spiral.
- iv. The divided points are control points of the NURBS curve and the NURBS interpolation curve is generated to be a spiral from the extremum point to the outer edges.
- v. Machining points are traced with the Runge-Kutta method by using Eqs. (3), (4), and (5).

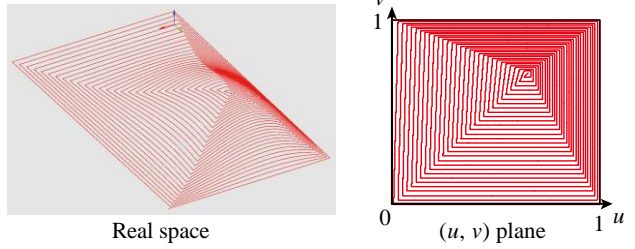


Figure 5. Result of tool path generation for rank 2

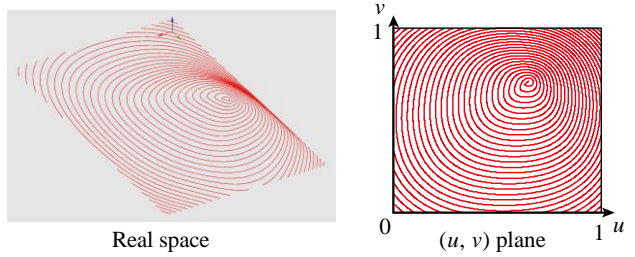


Figure 6. Result of tool path generation for rank 3

Figure 4 shows the result of the extremum search. The cross

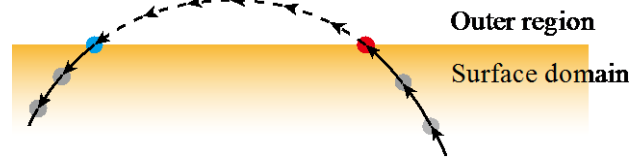


Figure 7. Treatment in the case outside the surface domain

point of the extremum search curves is the extreme point, which is adopted as the convergent point of the spiral path. Figures 5 and 6 show the results of the calculation of the spiral paths. We can confirm the generation of homogeneous tool paths that do not depend on the features of the machining surface. Figure 5 is rank 2 of the NURBS interpolation curve, and Figure 6 is rank 3. When the rank is 2, the NURBS curve is expressed as a polygonal curve. When the rank is 3, since the NURBS curve is a C2 continuous curve, the curve expands to the outer sides rather than to the position of the control points. In some cases, such as that shown in Figure 7, the curve exceeds the surface domain. In Figure 7, the calculation using the Runge-Kutta method is suspended, and the calculation is restarted when the trace point enters the surface domain from the outer side. The calculation time in both is approximately 7 seconds. Therefore, if using all CPU cores, the calculation time differs little from that of commercial CAM applications.

### 4. Conclusion

This study generated a homogeneous machining path without dependence on surface features. The results obtained are as follows.

- (1) By defining parametric curves described on the  $(u, v)$  plane, which contains the machining surface, homogeneous tool paths can be generated quickly.
- (2) The generated tool path changes according to the rank of the NURBS interpolation curve expressed on the  $(u, v)$  plane.

### References

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