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Biconic Zernike Surface Using a Cartesian Coordinate Machine and an Analytical Solution of the Tool Path

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Abstract

An physical solution for obtaining a biconic Zernike surface was developed in Cartesian coordinates. The tool path for fly cutting was calculated using contact conditions and geometric relations. The software for the tool path generated G-code from optical design parameters of the biconic Zernike surface. The G-code was uploaded to a four-axis machine constructed using a nanopositioning stage, air guides, kinematic balances, and environment controls. The stage was composed of XYZ linear transfer units and a C rotational unit. The moving parts of the linear transfer units were levitated using air guides and positioning accuracy was achieved using nano-feedback devices. The weight balance of the kinematic parts and air guides were studied in the design optimization. Our machine was operated under environment controls minimizing temperature variation, pneumatic fluctuation, and external vibration. Control factors of the axes were tuned to achieve 5 nm resolution. Our experiment was implemented to obtain a copper surface of a convex F-theta surface using fly cutting.

Zernike polynomials, biconic Zernike surface, F-theta lens, Fly-cutter, Air guide, Cartesian coordinate

1. Introduction

Zernike polynomials are a theoretical optical surface whose characteristics are irregular, non-linear, complex and non-rotationally symmetric. It is still difficult to obtain a Zernike surface from Zernike polynomials, but the Zernike surface has prospective applications such as bio-photonics, scanning, and laser printing, as well as 3D printing [1,2]. The Zernike surface has typically been obtained using a five-axis ultra-precision machine. Tool path for the Zernike surface can be generated using freeform surfacing [3]. Zernike surfaces are obtained using a high-precision machine and tool-path generation. Commercial CAD/CAM software for designing optics (i.e., CODE V) generates CNC code with optical parameters [4]. Diamond turning machines (DTMs) utilize five axes and ultra-precision to achieve the Zernike surface [5]. Slow tool servos are applied to compensate and correct surface error [6].

In this study, analytical and physical solutions of Zernike surfaces were considered to reduce numerical error. A software was used to generate G-code from biconic Zernike constants using the analytical solution. The machine had three axes of Cartesian coordinate and a fly-cutter. To achieve ultra-precision, air guides and nano-feedback devices were installed on linear transfer units. Design optimization was performed for weight balance and air guides. A test was implemented to obtain a convex F-theta surface using oxygen-free copper.

2. Analytical solution of the biconic Zernike surface

A Zernike surface is originally represented in high-order and spherical polar coordinates, making it difficult to obtain an analytical solution. The Zernike surface can be estimated using various polynomials; biconic Zernike is one of the approximated formulations [7]. The biconic Zernike surface is a tridimensional function described in Cartesian coordinate as follows:

$$z(x,y) = \frac{c_x x^2 + c_y y^2}{1+\sqrt{1-(1+k_x)c_x^2x^2-(1+k_y)c_y^2y^2}} + \sum_{i=1}^{16}\alpha_i x^i + \sum_{i=1}^{16}\beta_i y^i + \sum_{i=1}^{N} A_i Z^i(\rho,\varphi) \quad (1)$$

where x and y are planar coordinates of the biconic Zernike surface, z is the surface height, c denotes curvatures, k represents conic constants, α and β are aspheric constants, and A is ignored.

A fly-cutter rotated on the y_t axis has a torus shape; hence, the tool surface can be represented using following equation.

$$\left[R-\sqrt{(z-z_t)^2+(x-x_t)^2}\right]^2 + (y-y_t)^2 = r^2 \quad (2)$$

where R is the wheel radius, r is the tool radius, and (x_t, y_t, z_t) is center of the tool [8].

Normal vectors of both surfaces are equal at the contact point, and thus, both gradients should be calculated. First, in equation (1), terms within the square root are replaced with u. Then, the gradient of the biconic Zernike surface can be derived.

$$\left(\frac{\partial z}{\partial x}\right)_B = c_x x\frac{2\sqrt{u}(1+\sqrt{u})+(c_x x^2+c_y y^2)(1+k_x)c_x}{\sqrt{u}(1+\sqrt{u})^2} + \sum_{i=1}^{16}\alpha_i i x^{i-1} \quad (3)$$

$$\left(\frac{\partial z}{\partial y}\right)_B = c_y y\frac{2\sqrt{u}(1+\sqrt{u})+(c_x x^2+c_y y^2)(1+k_y)c_y}{\sqrt{u}(1+\sqrt{u})^2} + \sum_{i=1}^{16}\beta_i i y^{i-1} \quad (4)$$

The gradient of the fly-cutter can be represented in equations (5) and (6) using equation (2).

$$\left(\frac{\partial z}{\partial x}\right)_f = \frac{x - x_t}{z - z_t} \quad (5)$$

$$\left(\frac{\partial z}{\partial y}\right)_f = \frac{(y - y_t)\sqrt{(z - z_t)^2 + (x - x_t)^2}}{(z - z_t)(\sqrt{(z - z_t)^2 + (x - x_t)^2} - R)} \quad (6)$$

Both gradients are equal; thus, equations (7) and (8) can be obtained using equations (5) and (6).

$$(x - x_t) = \left(\frac{\partial z}{\partial x}\right)_B (z - z_t) \quad (7)$$

$$(y - y_t) = \left(\frac{\partial z}{\partial y}\right)_B \left[(z - z_t) - \frac{R}{\sqrt{\left(\frac{\partial z}{\partial x}\right)_B^2 + 1}}\right] \quad (8)$$

Equations (7) and (8) can substitute *x* and *y* terms in equation (2), and then z_t can be obtained as follows.

$$(z - z_t) = \frac{R}{\sqrt{\left(\frac{\partial z}{\partial x}\right)_B^2 + 1}} \pm \frac{r}{\sqrt{\left(\frac{\partial z}{\partial x}\right)_B^2 + \left(\frac{\partial z}{\partial y}\right)_B^2 + 1}} = \Delta z \quad (9)$$

The values for x_t and y_t can also be calculated by applying equation (9) to equations (10) and (11).

$$(x - x_t) = \left(\frac{\partial z}{\partial x}\right)_B \Delta z \quad (10)$$

$$(y - y_t) = \pm \frac{r\left(\frac{\partial z}{\partial y}\right)_B}{\sqrt{\left(\frac{\partial z}{\partial x}\right)_B^2 + \left(\frac{\partial z}{\partial y}\right)_B^2 + 1}} \quad (11)$$

When the tool path is generated, x and y are represented on a grid, and then z is calculated using the biconic Zernike surface equation (1). Furthermore, the gradient can also be calculated using (3) and (4). Therefore, the tool position can be calculated using equations (9) - (11), which are analytical and physical solutions.

3. Experiment and results

The analytical solution was incorporated into software for G-code generation. Fig. 1 shows a screen shot of the software that generated the G-code from design parameters of the biconic Zernike surface. In the experiment, the size of the convex surface was 14 x 100 mm^2.

The G-code was uploaded a four-axis ultra-precision machine. The machine had XYZC axes and XYZ axes, linear transfer units, were driven by the G-code during fly cutting. Air guides were applied to the linear transfer units for friction minimization and nano-feedback devices were applied to control the axes (PMDi Polaris/Magnescale BS78). Before installing the linear transfer units onto the base frame, the shape of the base frame, the arrangement of the axes, and weight balance were evaluated using ANSYS. The optimum design was determined by maximum compliance of the frequency response.

The balance and stiffness of the air guides were simulated using the orifice size and the locations, depth and number of air ports. The fluctuation of the pneumatic pressure affected the stiffness and the air gap; thus, a double pneumatic regulator was attached to the pneumatic suppy line. The temperature and humidity of the pneumatic pressure were maintained using a thermal-air dryer. The ultra-precision machine was enclosed in a chamber that controls humidity and temperature. The temperature variation was ±0.15^{o}C/day. The vibration from the floor was eliminated using pneumatic isolators, and the planar level was controlled with ±10μm accuracy. An R-bite type fly-cutter was fixed on a high-speed air spindle. After assembly, the accuracy and straightness of the linear transfer units were less than 10 nm and 60 nm, as measured by laser-interferometer.

The test sample was made of oxygen-free copper and the biconic Zernike surface was achieved by fly cutting, as shown in Fig. 2. The surface accuracy after processing was measured using a contact probe (Taylor Hobson PGI 840) with an accuracy of ±2.5μm on the major axis.

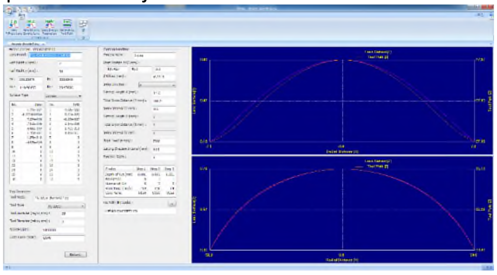

Figure 1. Software used to generate the tool path based on the analytical solution

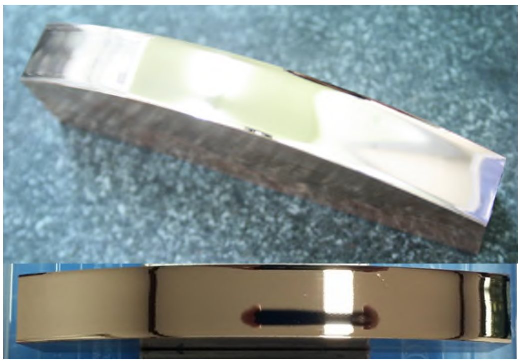

Figure 2. Biconic Zernike surface after fly cutting

4. Conclusion

The tool path of a biconic Zernike surface was achieved using analytical and physical solutions in Cartesian coordinates. The solution was derived from geometric relations of a fly-cutter and the biconoc Zernike surface, and it was implemented into a software that directly generated G-code from surface design parameters. The G-code was then uploaded into an ultra-precision machine, which operated with nano precision and strict environment controls. The surface accuracy after fly cutting was was ±2.5 μm.

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