euspen's 24th International Conference & Exhibition, Dublin, IE, June 2024

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Robust system performance analysis for viscoelastic damper materials

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Abstract

High precision positioning of mechatronic components in industrial applications requires a robust system design. Resonances within mechanical structures affect the stability of position control loops. To achieve high positioning bandwidth, passive damping strategies such as relative or tuned-mass dampers are used. Passive dampers are often made of viscoelastic materials such as elastomers, which are sensitive to temperature and vary in manufacturing tolerances. Due to the large uncertainties of viscoelastic materials system performance needs to be guaranteed for all possible configurations. In this contribution we present the use of the µ-analysis for viscoelastic materials to evaluate robust system performance for geometrical, material and temperature tolerances.

The system performance can be evaluated by analyzing the sensitivity function of the closed loop system within the µ-analysis. The used dynamic model is based on a finite element model, where viscoelastic damper models are added by a feedback loop. Tolerances such as variations in Young's modulus are also represented by a feedback loop, typical structure for the µ-analysis. The frequency dependent Young's modulus of viscoelastic materials is approximated by a linear transfer function based on a dynamic mechanical analysis. For elastomers the Young's modulus mainly varies in amplitude and shifts in frequency due to temperature. There is a direct relation between temperature change and frequency shift of the Young's modulus. Considering the temperature change instead of a stiffness change reduces conservatism in the performance analysis. For example, using several elastomers and considering only stiffness variations leads to the possibility that one elastomer sees an increase and the other a decrease in stiffness as a worst-case scenario, a non-physical behavior. µ-analysis with reduced conservatism improves design costs of mechatronic components and gives a robustness guarantee instead of a time-consuming Monte-Carlo simulation.

Robust control, Viscoelastic material, µ-Analysis

1. Introduction

Neglected component tolerances in system design of high precision mechatronic systems can lead to system performance losses during qualification of series production. Consequently, significant costs occur for solving the out of specification situation. Therefore, a robust design of mechatronic systems is necessary, where the tolerance effects on system performance are investigated. For example in high precision positioning systems passive dampers are used to prevent instabilities of the control loop due to undamped structural dynamics. These dampers are often made of viscoelastic materials, which are strongly sensitive to temperature and the manufacturing process such as geometry and the material properties. Occasionally, a Monte-Carlo simulation is used for investigating the system performance with tolerances. However, in high precision mechatronic systems there are several tolerances, where you cannot guarantee a robust performance within a Monte-Carlo simulation in finite time due to the large number of combinations. For that reason, we use the µ-analysis [1], an optimization-based approach to investigate robust performance of a controlled system. This approach can guarantee robustness for a large number of tolerances. For that reason, we present in this paper the use of the µ-analysis for large mechanical structures with viscoelastic damper materials. The temperature, geometrical and material tolerances are considered within the µ-analysis.

The paper is organized as follows: First, we show a structural dynamic model generation out of a finite element solver. Then, we describe how to add viscoelastic damping to that model. A dynamic model for the Young's modulus is derived in the time domain, where a description of the temperature, geometrical and material tolerances is included. Based on the parametric model an overall state-space model is presented for the use in the µ-analysis. Then, conditions for robust performance based on the µ-analysis are defined. Finally, the approach is applied to a three mass-spring example.

2. Viscoelastic damper modeling

Due to the small movements in high precision position control the mechanics can be described by the linear elasticity theory. The corresponding analytical partial differential equations are approximated by the finite element method (FEM). In order to analyze the dynamic behavior of the viscoelastic dampers, we extract a state space description from the FE solvers such as *MSC Nastran* or *Ansys*. Because of the large number of degree of freedoms (DOF) only the information of the modal analysis is used. To also consider viscous dampers, the normalization of the eigenvectors needs to be considered. In a FE-solver the modal analysis is performed by the created model based on the mass M, stiffness K and viscous damping D matricesThe equations of motion can be represented by [2]

$$\underbrace{\begin{bmatrix} D & M \\ M & 0 \end{bmatrix}}_{\tilde{M}} \begin{bmatrix} \dot{x}_{\mathrm{FE}} \\ \ddot{x}_{\mathrm{FE}} \end{bmatrix} + \underbrace{\begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}}_{\tilde{K}} \underbrace{\begin{bmatrix} x_{\mathrm{FE}} \\ \dot{x}_{\mathrm{FE}} \end{bmatrix}}_{z_{\mathrm{FE}}} = \underbrace{\begin{bmatrix} f_{\mathrm{FE}} \\ 0 \end{bmatrix}}_{f}, \qquad (1)$$

where x_{FE} is the displacement and f the force at each node in all considered DOFs. The ordinary differential equations can be transformed into modal space by $z_{\mathrm{FE}} = Q q_{\mathrm{FE}}$, where Q contains the eigenvectors of eq. (1). Due to symmetry eigenvectors are normalized to $Q^{\mathrm{T}}\widetilde{M}Q = I$, where I corresponds to the unity matrix. Considering this normalization in the modal analysis of the FE-solver a state space in modal representation can be generated based on calculated eigenvectors within the FE-solver

$$\begin{aligned} \Sigma_{\mathrm{FE}}: \dot{q}_{\mathrm{FE}} &= -\underbrace{Q^{\mathrm{T}}\widetilde{K}Q}_{\Lambda}\, q_{\mathrm{FE}} + Q^{\mathrm{T}} f \\ z_{\mathrm{FE}} &= Q q_{\mathrm{FE}}, \end{aligned} \tag{2}$$

where q_{FE} denotes the state in modal coordinates. Due to the frequency-dependent Young's modulus of viscoelastic materials the damping behavior cannot be considered in the modal analysis. However, the damping can be added by a dynamical model in a feedback loop afterwards, which is mechanically comparable to a parallel frequency-dependent spring. For that reason, the in- and outputs (IOs) of the Σ_{FE} state space are modified by only selecting the IOs with respect to the viscoelastic dampers and to the position control

$$\begin{aligned} \begin{bmatrix} x_{\mathrm{M}} \\ x_{\mathrm{VE}} \end{bmatrix} &= \begin{bmatrix} C_{\mathrm{M}} \\ C_{\mathrm{VE}} \end{bmatrix} z_{\mathrm{FE}} \\ f &= \begin{bmatrix} B_{\mathrm{A}} & B_{\mathrm{VE}} \end{bmatrix} \begin{bmatrix} f_{\mathrm{A}} \\ f_{\mathrm{VE}} \end{bmatrix}, \end{aligned} \tag{3}$$

where C_{M}, C_{VE} selects the displacements and F_{A}, F_{viscIO} the forces for the position control and the viscoelastic damper element nodes, respectively. The FE model already contains the stiffness of the viscoelastic damper. Therefore, only the stiffness change over frequency is described by an additional force to the FE model

$$\mathbf{f}_{\mathrm{VE}}(s) = \left(\frac{\mathbf{e}(s)}{\mathbf{e}(0)} - 1\right) K_{\mathrm{VE}}\, \mathbf{x}_{\mathrm{VE}}(s), \tag{4}$$

where s represents the variable of the Laplace transformation, bold letters correspond to the laplace transformed variable and K_{VE} is the stiffness matrix of the viscoelastic damper. The viscoelastic material behavior is described by a frequency-dependent Young's modulus e. Based on the introduced equations an overall state space Σ_{VE} can be derived

$$\begin{aligned} \Sigma_{\mathrm{VE}}: \dot{q}_{\mathrm{FE}} = &-(\Lambda + Q^{\mathrm{T}} B_{\mathrm{VE}} K_{\mathrm{VE}} C_{\mathrm{VE}} Q) q_{\mathrm{FE}} \\ &+ Q^{\mathrm{T}} B_{\mathrm{A}} f_{\mathrm{A}} + \frac{1}{\mathbf{e}(0)} Q^{\mathrm{T}} B_{\mathrm{VE}} f_{\mathrm{e}} \\ x_{\mathrm{M}} = &\; C_{\mathrm{M}} Q q_{\mathrm{FE}} \\ x_{\mathrm{e}} = &\; K_{\mathrm{VE}} C_{\mathrm{VE}} Q\, q_{\mathrm{FE}}. \end{aligned} \tag{5}$$

in Figure 1 the feedback loop with respect to the Young's modulus is represented.

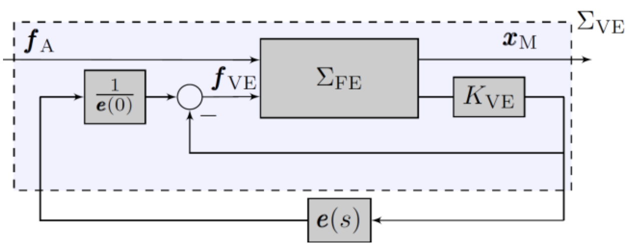

Figure 1. Considering viscoelastic materials in dynamic models by a feedback of a frequency-dependent Young's modulus.

The dynamic behavior of the Young's modulus of the viscoelastic material is often determined by a FRF measurement, also known as the dynamic mechanical analysis (DMA). In order to analyze the impact on the position control, a dynamic model is fitted into the measurement data. The number of poles and zeros of the transfer functions are chosen to be equal, to obtain a proper transfer function. Here, the fitting can also be interpreted as a parameter estimation of the generalized Maxwell model [3], a physical model. Then, the frequency-dependent Young's modulus is described by

$$\mathbf{e}(s) = g \prod_{i=0}^{n-1} \frac{(s+z_i)}{(s+p_i)}, \tag{6}$$

where p_i and z_i describe the poles and zeros, and g a scaling factor. Moreover, the Young's modulus of viscoelastic materials significantly depends on temperature T, which can be modeled by frequency scaling of the dynamic Young's modulus. For viscoelastic material the Williams-Landel Ferry model is a state of the art approach for shifting frequencies of the Young's modulus [4]

$$\begin{aligned} \mathbf{e}_{\mathrm{T}}(j\omega, T) &= \mathbf{e}(ja(T)\omega) \\ \log_{10}(a) &= -\frac{c_1(T-T_0)}{c_2 + (T-T_0)}, \end{aligned} \tag{7}$$

where a_{T} is the frequency scaling factor, c_1, c_2 are viscoelastic material parameters, T_0 is the reference temperature of the measured Young's modulus FRF and j is the complex number. In order to study the temperature-dependent control performance, we need a dynamic model, that represents the frequency shift as in eq. (7). A frequency shift can be realized by scaling the poles and zeros of the Young's modulus in eq. (6). For the μ-analysis the Young's modulus is described in a modal state space representation

$$\begin{aligned} \dot{q}_{\mathrm{e}} &= \underbrace{\begin{bmatrix} -p_1 & 0 & \dots & \dots & 0 \\ 0 & -p_2 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & -p_{n-1} \end{bmatrix}}_{A_e} q_{\mathrm{e}} + \underbrace{\begin{bmatrix} 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix}}_{b_e} x_{\mathrm{e}} \\ f_{\mathrm{e}} &= -g \underbrace{\begin{bmatrix} r_0 & \dots & r_{n-1} \end{bmatrix}}_{r_e^T} q_{\mathrm{e}} + g x_{\mathrm{e}}, \end{aligned} \tag{8}$$

where r_{e} describes the residual based on the zeros and poles of the transfer function. Now, the system matrix and the residuals are scaled to

$$\begin{aligned} \dot{q}_{\mathrm{e}} &= a(T) A_{\mathrm{e}} q_{\mathrm{e}} + b_{\mathrm{e}} x_{\mathrm{e}} \\ f_e &= -a(T) g r_{\mathrm{e}}^{\mathrm{T}} q_{\mathrm{e}} + g x_{\mathrm{e}}. \end{aligned} \tag{9}$$

2.1. Uncertainty modeling

In this paper the robust system performance of a position controlled system is analyzed by the μ-analysis. In order to use the corresponding framework, uncertainties are represented in a feedback structure as depicted in Figure 2. The mechanical system from actuator forces to position measurements, also called plant, are combined with two feedback loops for the position controller and the uncertainties. The Δ block represents the tolerances of the viscoelastic material, where we consider structured uncertainties, diagonal blocks for each tolerance scaled to $-1\dots1$. Plant and Controller combined to the nominal system Σ_N.

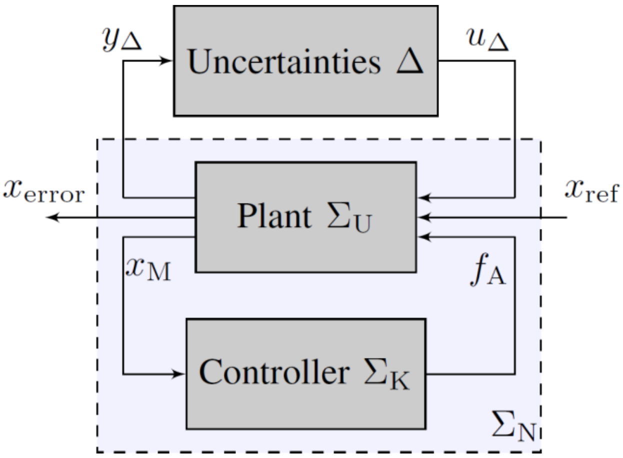

Figure 2. Generalized plant representation of the mechatronic system for the µ-analysis framework**.**

The uncertain parameters of the viscoelastic Young's modulus need to be described in a relative representation to receive a uncertainty between -1 and 1. For the temperature we can derive the interval of the shift factor by

$$\begin{aligned} a_\Delta &= a_0(1+\Delta_{\mathrm{T}} a_{\mathrm{r}}),\ \Delta_{\mathrm{T}} = -1...1 \\ a_0 &= \tfrac{a(T_{\max})+a(T_{\min})}{2} \\ a_{\mathrm{r}} &= \tfrac{a(T_{\max})-a(T_{\min})}{a(T_{\max})+a(T_{\min})}, \end{aligned} \tag{10}$$

where a_0 corresponds to the mean value of the interval, a_{r} scales uncertainty feedback to -1...1 and $T_{\min}$, $T_{\max}$ are the minimum and maximum temperature of the tolerances. The geometrical and material uncertainties are considered in the gain factor of the Young's modulus

$$g_\Delta = g_0\left(1+\Delta_{\mathrm{g}} g_{\mathrm{r}}\right), \tag{11}$$

where g_0 corresponds to the center of the interval and g_r the relative change of the scaling factor. Based on the relative representation the state space of the Young's modulus can be formulated in a structure, where the introduced uncertainties are described within a feedback loop

$$\begin{aligned} \dot{q}_{\mathrm{e}} &= a_0 A_{\mathrm{e}} q_{\mathrm{e}} + b_{\mathrm{e}} x_{\mathrm{e}} + [I \quad 0] u_{\mathrm{T}} \\ f_{\mathrm{e}} &= -a_0 g_0 r_{\mathrm{e}}^{\mathrm{T}} q_{\mathrm{e}} + g_0 x_{\mathrm{e}} + [0 \quad g_0 I] u_{\mathrm{T}} + g_{\mathrm{r}} u_{\mathrm{g}} \\ y_{\mathrm{T}} &= \begin{bmatrix} a_{\mathrm{r}} a_0 A_{\mathrm{e}} \\ -a_0 r_{\mathrm{e}}^{\mathrm{T}} \end{bmatrix} q_{\mathrm{e}} \\ u_{\mathrm{T}} &= \Delta_{\mathrm{T}} y_{\mathrm{T}} \\ y_{\mathrm{g}} &= -a_0 g_0 r_{\mathrm{e}}^{\mathrm{T}} q_{\mathrm{e}} + g_0 x_{\mathrm{e}} + [0 \quad g_0 I] u_{\mathrm{T}} \\ u_{\mathrm{g}} &= \Delta_{\mathrm{g}} y_{\mathrm{g}}. \end{aligned} \tag{12}$$

Combining eq. (3) and eq. (6) results in an overall state space of the mechatronic system with uncertainties Σ_{U}, which can be used for the μ-analysis. The nominal model is based on the closed-loop system without uncertainties. A typical position controller in mechantronic system is based on a proportional-integral-derivative (PID) controller $\mathbf{c}(s)$. The feedback law is defined by

$$\mathbf{f}_{\mathrm{A}}(s) = \mathbf{c}(s)\mathbf{x}_{\mathrm{error}}(s) = \mathbf{c}(s)\underbrace{\left(\mathbf{x}_{\mathrm{M}}(s) - \mathbf{x}_{\mathrm{ref}}(s)\right)}_{\mathbf{x}_{\mathrm{error}}(s)}, \tag{13}$$

x_{ref} defines the position reference. A parallel PID structure is used and for the derivative part a pole is added to not amplify high frequency flexible modes, which can lead to instabilities. Then, we obaint the following controller transfer function

$$\mathbf{c}(s) = \frac{\mathbf{f}_{\mathrm{A}}(s)}{\mathbf{x}_{\mathrm{error}}(s)} = k_{\mathrm{P}} + k_{\mathrm{I}}\frac{1}{s} + k_{\mathrm{D}} s \frac{1}{1+\tau_{\mathrm{d}} s}, \tag{14}$$

where k_{P}, k_{I} and k_{D} are the controller gains and τ_{d} the roll-off time constant for the derivative part. In order to combine the controller in a state space model, the transfer function is reformulated in the time domain by

$$\begin{aligned} \Sigma_{\mathrm{K}}\colon \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ -\frac{k_{\mathrm{P}}}{\tau_{\mathrm{d}}^2} & -\frac{1}{\tau_{\mathrm{d}}} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -\left(\frac{k_{\mathrm{P}}}{\tau_{\mathrm{d}}} + \frac{k_{\mathrm{D}}}{\tau_{\mathrm{d}}^2}\right) \end{bmatrix} x_{\mathrm{error}} \\ f_{\mathrm{A}} &= \left[\left(k_{\mathrm{I}} + \frac{k_{\mathrm{P}}}{\tau_{\mathrm{d}}}\right) \quad 1\right] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \left(k_{\mathrm{P}} + \frac{k_{\mathrm{D}}}{\tau_{\mathrm{d}}}\right) x_{\mathrm{error}}. \end{aligned} \tag{15}$$

From eq. (12), eq. (13) and eq. (15) we obtain an overall state space for our nominal model Σ_{N} with in- and outputs for the uncertainties.

2.2. Robustness analysis

For analyzing the robust performance, we use the μ-analysis, a frequency domain approach. Therefore, from the Σ_{N} state space a transfer function $\mathbf{N}(s) = C_{\mathrm{N}}(sI - A)^{-1} B_{\mathrm{N}} + D_{\mathrm{N}}$ is determined. The transfer matrix can be divided into the following representation

$$\begin{bmatrix} \mathbf{y}_\Delta \\ \mathbf{x}_{\mathrm{error}} \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{11}(s) & \mathbf{N}_{11}(s) \\ \mathbf{N}_{21}(s) & \mathbf{N}_{22}(s) \end{bmatrix} \begin{bmatrix} \mathbf{u}_\Delta \\ \mathbf{x}_{\mathrm{ref}} \end{bmatrix}, \tag{16}$$

where $\mathbf{y}_\Delta$,$\mathbf{u}_\Delta$ are the in- and outputs for the uncertainties. For studying the robust performance, the output sensitivity is used, the transfer from reference to servo error signal, a quantity for the distance from open-loop function to the critical point of the Nyquist stability criterion. The structured singular value, also known as μ is defined by

$$\mu(\mathbf{M}) = \frac{1}{\inf\{\sigma_{\max}(\Delta) | \det(I - \mathbf{M}\Delta) = 0\}}, \tag{17}$$

where $\sigma_{\max}$ denotes the maximum singular value of the block matrix Δ. If the nominal system is stable and $\mu(\mathbf{N}_{11}) < 1$, the system is robust stable. Performance criteria can be considered by uncertainties as well. To fulfill feedback values < 1, the corresponding sensitivity outputs need to be scaled. As a performance criterion we claim a maximum output sensitivity S_{max} Therefore, we get a scaling factor $W_{\mathrm{o}} = \frac{1}{S_{max}}$. For the μ-analysis a new transfer matrix is defined by

$$\mathbf{O}(s) = \begin{bmatrix} \mathbf{N}_{11}(s) & \mathbf{N}_{11}(s) \\ \mathbf{N}_{21}(s) & W_{\mathrm{o}}\mathbf{N}_{22}(s) \end{bmatrix} \tag{18}$$

Robust performance is achieved, if $\mu(\mathbf{O}) < 1$.

3. Example

The robustness analysis of viscoelastic materials for temperature, geometrical and material tolerances is applied for a three mass-spring system to illustrate the method. In Figure 3 the spring-mass system is depicted. A mass m is controlled by a actuator with a clearly smaller mass m_{A} and stiffness k_{A}. Then, the corresponding resonance frequency can be approximated by $\omega_{\mathrm{A}}^2 = \frac{k_{\mathrm{A}}}{m_{\mathrm{A}}}$. The resonance frequency is significantly higher than the cross-over frequency of the control loop, but it causes instabilities without damping.

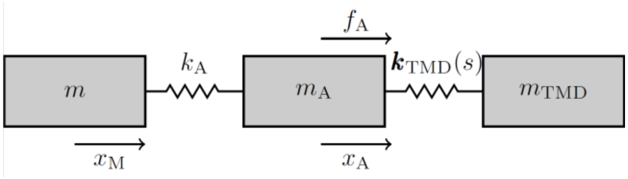

Figure 3. Depiction of a three mass-spring system example to analyze robust performance with viscoelastic materials.

For that reason, the actuator is damped by a tuned-mass-damper (TMD), which is modeled by a mass m_{TMD} and a frequency dependent stiffness $\mathbf{k}_{\mathrm{TMD}}(s)$. The TMD mass is also by a factor ten smaller and the stiffness is designed in a way that the actuator resonance is sufficient damped. The viscoelastic damping effect is modeled by the frequency dependent Young's modulus. The corresponding stiffness and mass matrix of the example is described in the representation from eq. (1), where the frequency-dependent stiffness $\mathbf{k}_{\mathrm{TMD}}$ is chosen at 0 Hz. Moreover, the used normalized Young's modulus for the frequency dependency is depicted in Figure 4. A transfer function with three poles and zero is fitted into a data set of an elastomer by the least square approach.

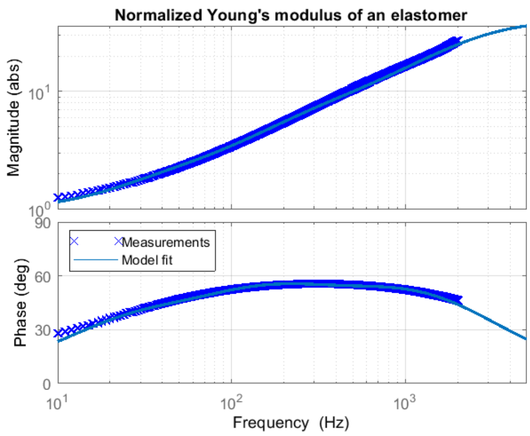

Figure 4. Comparison of the measured Young's modulus of an elastomer and the fitted transfer function. There is a significant deviation between 1 Hz and 10 Hz due to the limited number of poles and zeros to three. In our example the interesting damping frequency is larger 100 Hz.

The position controller is tuned for a cross-over frequency of 160 Hz. For the geometrical and material tolerances we assume 20 % deviation at the maximum. Moreover, two temperature intervals for a robust (22 °C … 27 °C) and non-robust case (22 °C ... 37 °C) are considered. The calculated μ-values over frequency are depicted in Figure 5. For the robust case the µ-value is clearly smaller than one. Worst case parameters are at 20 % for geometrical and material deviations and at 27 °C. For an interval 22 °C … 37 °C the robust performance is violated around the resonance frequency of the actuator, µ-values clearly larger 1. The worst-case parameters are at -20% geometrical and material deviations, and at 37 °C. As a result, the stiffness for the TMD is as weak as possible for the non-robust case. In Figure 6 impact on output sensitivities are depicted of the nominal, non-robust and robust case.

4. Summary

In this paper a method is shown to analyze robust performance of mechatronic systems with viscoelastic damper materials. A

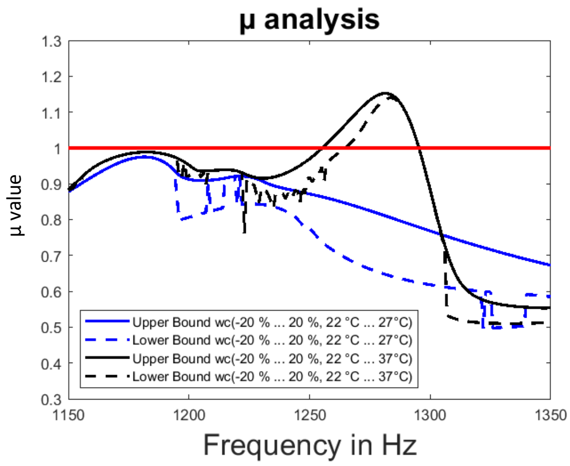

Figure 5. Example results of the µ-analysis for a non-robust and robust performance. An interval of the µ value based on lower and upper bound are given due to complex calculation of µ. Increasing the temperature tolerances leads to a performance violation.

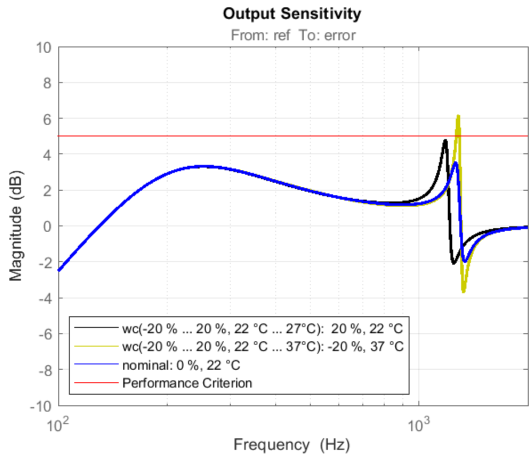

Figure 6. Representation of the 5 dB robust performance criterion, the output sensitivity with worst case parameters, for the nominal, robust and non robust case.

dynamic model is fitted into measured frequency response functions of the Young's modulus. The dynamics model can be adapted to different temperatures and geometrical and material tolerances. In order to study the impact on position control of mechatronic system, the robustness can be investigated by the µ-analysis. The presented method allows an analysis of large FE models with viscoelastic damping materials. This cannot be considered in the FE-solvers by default. Moreover, temperature effects can be covered within the robustness analysis by taking in account of the frequency-dependent effect of the Young's modulus. Based on that conservatism can be taken out from the µ-analysis by using temperature tolerances instead of pure stiffness variation.

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